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Strings at the tachyonic vacuum

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Abstract: We study the world-volume effective action of Dp-brane at the tachyonic vacuum which is equivalent to the zero tension limit. Using the Hamiltonian formalism we discuss the algebra of constraints and show that there is a non-trivial ideal of the algebra which corresponds to Virasoro like constraints. The Lagrangian treatment of the model is also considered. For the gauge fixed theory we construct the important subset of classical solutions which is equivalent to the string theory solutions in conformal gauge. We speculate on a possible quantization of the system. At the end a brief discussion of different background fields and fluctuations around the tachyonic vacuum is presented.

Keywords: Bosonic Strings, D-branes.

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Contents

1. Introduction and motivation

The problem of tachyon condensation is an old subject in the context of the string theory [1]. Recently it has been conjectured that the tachyonic vacuum in open string theory on the D-brane describes the closed string vacuum without D-branes and that various soliton solutions of the theory describe D-branes of lower dimension [2]. This conjecture has been supported by number calculations within the first and second quantized string theory.

One can get some insight into the problem by considering the world-volume effective action [3] which describes the D-brane around the tachyonic vacuum. Recently there has been some effort directed towards identifying of string-like classical solutions whose tension matches that of fundamental string [4]–[8]. In fact the D-brane action at the tachyonic vacuum is equivalent to the zero tension limit of the D-brane action. The zero tension limit was with a different motivation studied previously in [9]–[11] and a string picture was obtained there as well.

In this short note we would like to revise and clarify certain arguments from [9]–[11] in light of the new motivation. Unlike [7, 8] we do not start from the gauge fixed model. The ultimate goal is to relate two different theories (zero tension D-brane theory and string theory) to each other. These two theories have different gauge symmetries and are unrelated a priori. We study the algebra of constraints of D-brane theory at the tachyonic vacuum in detail. It turns out that fundamental string-like Hamiltonian constraints generate a subalgebra (in fact an ideal) of the full algebra of the model at the tachyonic vacuum and that there is natural embedding of this subalgebra provided by the "electric flux". We hope that the present analysis will clarify the general situation and as well as explore the relation between the static gauge results [7, 8] and the general situation. Our formalism is Poincaré covariant, i.e. we avoid the gauge fixing which breaks the Poincaré invariance (i.e static gauge).

In addition we study the lagrangian which describes the dynamics of D-brane at the tachyonic vacuum. In the set of solutions of the classical equations of the gauge fixed model we identify the subset of solutions that correspond to string theory solutions in conformal gauge. We argue that the model might be consistently quantized if we regard the electric and magnetic fields as a type of background fields.

At the tachyonic vacuum the RR background fields decouple completely from the dynamics since the Wess-Zumino couplings are proportional to the derivative of the tachyon field. This is natural since one expects that the tachyonic vacuum is equivalent to the closed string vacuum without D-branes. However the fluctuations around the vacuum should describe the D-branes of lower dimension. Therefore it is natural to study these fluctuation within the present framework. At the end of the paper we thus briefly discuss small fluctuation around the tachyonic vacuum.

The paper is organized as follows: in section 2 we study the algebra of constraints within the hamiltonian formalism. Section 3 is devoted to the lagrangian treatment of the model and to the classical solutions of the gauge fixed theory. In the section 4 the role of the background antisymmetric tensor fields and small fluctuations around the vacuum are discussed. In the last section 5 we summarize the results and propose some future research.

2. Hamiltonian treatment of BI theory

In this section we study the algebra of constraints in detail and find that the string-like constraints generate an ideal inside the full algebra.

Let us start by considering the effective D-brane action [3] with constant tachyon field T

$$S = T_p V(T) \int d^{p+1}x \sqrt{-\det(\gamma_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})}\,, \tag{2.1}$$

where $\gamma_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}$ is the pullback of the space-time metric, $F_{\alpha\beta} = \partial_{[\alpha} A_{\beta]}$ is the field strength for the U(1) gauge field A_α and $V(T)$ is a tachyon potential. For the moment we ignore the antisymmetric background tensor fields (NS two-form and RR forms). In what follows we assume that the D-brane is closed (i.e. that the appropriate periodicity conditions on the fields are imposed) or that there is appropriate fall off of the field at the spatial infinity. This assumption is needed to avoid possible boundary terms.

Since (2.1) is a generally covariant system the naive hamiltonian vanishes. The constraints can be straightforwardly derived from the action (2.1) [9]

$$\mathcal{H}_a = P_\mu \partial_a X^\mu + \pi^b F_{ab}\,, \tag{2.2}$$

$$\mathcal{H} = P_\mu G^{\mu\nu} P_\nu + \frac{1}{(2\pi\alpha')^2}\pi^a \gamma_{ab}\pi^b + T_p^2 (V(T))^2 det(\gamma_{ab} + 2\pi\alpha' F_{ab})\,, \tag{2.3}$$

$$\mathcal{G} = \partial_a \pi^a\,, \qquad \pi_0 = 0\,, \tag{2.4}$$

where we use lower case latin letters for the spatial indices. There are $(p+3)$ constraints as there should be corresponding to the $(p+1)$ diffeomorphisms and U(1) symmetries. It is convenient to smear the constraints with test functions

$$\mathcal{H}_a[N^a] = \int d^p x\, N^a(x)\mathcal{H}_a(x)\,, \qquad \mathcal{H}[M] = \int d^p x M(x)\mathcal{H}(x)\,,$$
$$\mathcal{G}[\Lambda] = \int d^p x \Lambda(x)\mathcal{G}(x) \tag{2.5}$$

where N^a is a p-dimensional vector, Λ is scalar and M is scalar density of weight minus one. M has this weight because the constraint (2.3) transforms as a scalar density of weight two and we would like to write the Hamiltonian as a sum of the constraints smeared by test functions. In analogy with general relativity we call N^a and M shift vector and lapse function, respectively. As usual for gauge theories we can identify Λ with the zero component of the gauge vector potential. The constraints obey an algebra whose non-zero brackets are

$$\{\mathcal{H}_a[N^a], \mathcal{H}_b[M^b]\} = \mathcal{H}_a[\mathcal{L}_{\vec{N}} M^a] + \mathcal{G}[N^a M^b F_{ba}]\,, \tag{2.6}$$

$$\{\mathcal{H}_a[N^a], \mathcal{H}[M]\} = \mathcal{H}[\mathcal{L}_{\vec{N}} M] + \mathcal{G}[2M\pi^a \gamma_{ab} N^b]\,, \tag{2.7}$$

$$\{\mathcal{H}[N], \mathcal{H}[M]\} = \mathcal{H}_a[4(\pi^a\pi^b + A^{(ab)})(N\partial_b M - M\partial_b N)]\,, \tag{2.8}$$

with the following notation

$$A^{ab} = T_p^2 V(T)^2 \frac{1}{(p-1)!}\epsilon^{aa_1\ldots a_{p-1}}\epsilon^{bb_1\ldots b_{p-1}}(\gamma_{a_1b_1}+F_{a_1b_1})\ldots(\gamma_{a_{p-1}b_{p-1}}+F_{a_{p-1}b_{p-1}}) \tag{2.9}$$

and $A^{(ab)} = (1/2)(A^{ab} + A^{ba})$. For the time being we drop a factor $(2\pi\alpha')$ to avoid cluttering the formulae. This factor can be easily restored in the final expressions. The full Hamiltonian is given by

$$H = \mathcal{H}_a[N^a] + \mathcal{H}[M] + \mathcal{G}[\Lambda]\,, \tag{2.10}$$

and provides the time evolution of the fields. The algebra (2.6)–(2.8) is not closed and has field dependent structure constants. As far as the authors are aware the classical BRST charge for this system has not been constructed. By analogy to the

p-brane case [12] one may expect that the BRST charge will have quite a high rank (maybe p as for p-branes [12]). Therefore it seems difficult to quantize this model from first principles.[1]

If the tachyon field is frozen at its minimum $T = T_0$ ($V(T_0) = 0$) the constraints are reduced to the following set

$$\mathcal{H}_a = P_\mu \partial_a X^\mu + \pi^b F_{ab} \,, \tag{2.11}$$

and

$$\mathcal{H} = P_\mu G^{\mu\nu} P_\nu + \frac{1}{(2\pi\alpha')^2} \pi^a \gamma_{ab} \pi^b \,, \tag{2.12}$$

together with Gauss' law $\partial_a \pi^a = 0$ and $\pi_0 = 0$. The constraints (2.11), (2.12) obey the following algebra

$$\begin{aligned}
\{\mathcal{H}_a[N^a], \mathcal{H}_b[M^b]\} &= \mathcal{H}_a[\mathcal{L}_{\vec{N}} M^a] + \mathcal{G}[N^a M^b F_{ba}] \,, &(2.13)\\
\{\mathcal{H}_a[N^a], \mathcal{H}[M]\} &= \mathcal{H}[\mathcal{L}_{\vec{N}} M] + \mathcal{G}[2M \pi^a \gamma_{ab} N^b] \,, &(2.14)\\
\{\mathcal{H}[N], \mathcal{H}[M]\} &= \mathcal{H}_a[4\pi^a \pi^b (N \partial_b M - M \partial_b N)] \,. &(2.15)
\end{aligned}$$

The algebra (2.13)–(2.15) is similar to that of the full theory (2.6)–(2.8). In fact at the tachyonic vacuum the system has the gauge symmetries of the full theory.[2] For the usual p-branes (i.e. with the gauge field in (2.1) equal to zero) this is not the case. The zero tension algebra for a p-brane has a completely different form from that of the full algebra.

There is one important difference in the field dependent structure constants of (2.15) and (2.8) which plays a key role in finding the string like subalgebra (2.21)–(2.23). Explicitly the right hand side of (2.15) is

$$\mathcal{H}_a[4\pi^a \pi^b (N \partial_b M - M \partial_b N)] = \int d^p x \, 4\pi^a \pi^b (N \partial_b M - M \partial_b N)(P_\mu \partial_a X^\mu + \pi^c F_{ac}) \,, \tag{2.16}$$

where the last term vanishes identically because of the antisymmetry of F_{ab}. Thus the right hand side of (2.16) leads to $P_\mu \pi^a \partial_a X^\mu = 0$ showing that $\pi^a \partial_a X^\mu = 0$ is a "preferred" direction on the world-volume.

As a direct result of (2.11), (2.12) there is not much dynamics for the U(1) degrees of freedom. For instance, the momenta π^a satisfy the following equations

$$\partial_a \pi^a = 0 \,, \qquad \dot{\pi}^a = \mathcal{L}_{\vec{N}} \pi^a \,, \tag{2.17}$$

[1]Even though we call the BI theory the effective theory it is still an unsettled question what is more fundamental, strings or D-branes. Thus a possible consistent quantization of the BI theory is an important issue.

[2]This is not the case in the $\alpha' \to \infty$ limit, where the gauge algebra has a different form.

and thus π^a completely decouples from the other fields, except from the lagrangian multiplier, $\vec{N}$, (the shift vector). Because of (2.16) one may decompose the constraints (2.11), (2.12) in the following fashion

$$\mathcal{H}_\pi = P_\mu \pi^a \partial_a X^\mu \, , \mathcal{H} = P_\mu G^{\mu\nu} P_\nu + \frac{1}{(2\pi\alpha')^2} \pi^a \gamma_{ab} \pi^b \, , \tag{2.18}$$

the other $(p-1)$ generators being

$$\mathcal{H}_a^\perp[\tilde{N}^a] = \int d^p x \, \tilde{N}^a (P_\mu \partial_a X^\mu + \pi^b F_{ab}) \, , \tag{2.19}$$

which generate the general coordinate transformations with parameter $\tilde{N}^a$ (see appendix A). The decomposition (2.18)-(2.19) may be thought of as a decomposition of the shift vector N^a along directions "parallel" and "orthogonal" to π^a

$$N^a = N\pi^a + \tilde{N}^a \, , \tag{2.20}$$

where N transforms as density of weight minus one. To really implement the concept "parallel" and "orthogonal" involves a metric, e.g., (the spatial part of) the induced metric, and leads to unwanted additional constraints. All we need is for $N\pi^a$ and $\tilde{N}^a$ to be linearly independent. The details of this decomposition are not essential in what follows, however. The appropriate decomposition of the constraint $\mathcal{H}_a$ can always be done locally. At the global level (2.20) may imply restrictions on the topology of the D-brane world-volume. In the following discussion we will disregard this potential complication.

For the given π^a we decompose the full set constrains as in (2.18), (2.19). The algebra of (2.18) is given by

$$\{\mathcal{H}_\pi[N], \mathcal{H}_\pi[M]\} = \mathcal{H}_\pi[\pi^a(N\partial_a M - M\partial_a N)] \, , \tag{2.21}$$

$$\{\mathcal{H}_\pi[N], \mathcal{H}[M]\} = \mathcal{H}[4\pi^a(N\partial_a M - M\partial_a N) + NM(\partial_a \pi^a)] \, , \tag{2.22}$$

$$\{\mathcal{H}[N], \mathcal{H}[M]\} = \mathcal{H}_\pi[4\pi^a(N\partial_a M - M\partial_a N)] \, . \tag{2.23}$$

This subalgebra closely resembles the algebra of constraints of the Nambu-Goto string. We have thus found a non trivial embedding of one algebra into another with field dependent structure constants.

Guided by this we introduce the following constraints

$$Q^\pm[N] = \mathcal{H}[N] \pm 2\mathcal{H}_\pi[N] = \int d^p x \, N(P_\mu \pm G_{\mu\sigma}\pi^a \partial_a X^\sigma) G^{\mu\nu} (P_\nu \pm G_{\nu\rho}\pi^b \partial_b X^\sigma) \, , \tag{2.24}$$

which are analogs of Virasoro constraints. In terms of the new constraints the algebra (2.13)–(2.15) becomes

$$\{Q^+[N], Q^+[M]\} = Q^+[8\pi^a(N\partial_a M - M\partial_a N)] \, , \tag{2.25}$$

$$\{Q^-[N], Q^-[M]\} = Q^-[8\pi^a(N\partial_a M - M\partial_a N)]\,, \tag{2.26}$$
$$\{Q^+[N], Q^-[M]\} = Q^+[NM\partial_a\pi^a] + Q^-[NM\partial_a\pi^a]\,, \tag{2.27}$$
$$\{\mathcal{H}_a^\perp[\tilde{N}^a], Q^\pm[M]\} = Q^\pm[\mathcal{L}_{\vec{\tilde{N}}}M] + \mathcal{G}[2\tilde{N}^a\pi^b M\gamma_{ab}]\,, \tag{2.28}$$
$$\{\mathcal{H}_a^\perp[\tilde{N}^a], \mathcal{H}_b^\perp[\tilde{M}^b]\} = \mathcal{H}_a^\perp[\mathcal{L}_{\vec{\tilde{N}}}\tilde{M}^a] + \mathcal{G}[\tilde{N}^a\tilde{M}^b F_{ba}]\,. \tag{2.29}$$

This algebra is thus exactly the same as (2.13)–(2.15) but written in a different form. The relation (2.27) can also be written as

$$\{Q^+[N], Q^-[M]\} = \mathcal{G}[2NM(P_\mu G^{\mu\nu}P_\nu + \pi^a\gamma_{ab}\pi^b)]\,. \tag{2.30}$$

The algebra (2.25)-(2.29) follows straightforwardly from the previous calculations. The only relation which needs checking is

$$\{\mathcal{H}_a[N^a], \mathcal{H}_\pi[M]\} = \mathcal{H}_\pi[\mathcal{L}_{\vec{N}}M]\,, \tag{2.31}$$

where there are no restrictions on N^a. The algebra (2.25)–(2.29) contains the Virasoro like generators $Q^\pm$ which together with Gauss law $\mathcal{G}$ generate an ideal of the full algebra. We know of no other nontrivial ideal of a gravity algebra with field dependent structure constants. It is unclear how this ideal is manifested in the BRST charge and other gauge theory quantities, but the existence of a nontrivial ideal may perhaps throw some light on the relation between theories with different gauge symmetries. We hope to return to this question elsewhere.

The algebra (2.25)–(2.29) is not closed and has field dependent structure constants. However since π^a decouples (see (2.17)) it is tempting to assume that Gauss' law holds strongly; $\partial_a\pi^a = 0$. Thus one may regard it as an "background field". Thus, considering a definite field configuration π^a with $\partial_a\pi^a = 0$ we may study the behavior of the system with this given "electric flux" π^a. Introducing a mode expansion for π^a and a mode expansion of the constraints $Q^\pm$, $\mathcal{H}_a^\perp$

$$\pi^a = \sum_{\vec{N}} \pi^a_{\vec{N}} e^{-i\vec{N}\vec{x}}\,, \quad L^\pm_{\vec{M}} = Q^\pm\left[\frac{1}{8}e^{-i\vec{M}\vec{x}}\right], \quad H_{a,\vec{N}} = \mathcal{H}_a^\perp[e^{-i\vec{N}\vec{x}}] \tag{2.32}$$

we get the following classical algebra

$$\{L^+_{\vec{N}}, L^+_{\vec{M}}\} = i\sum_{\vec{S}} \pi^a_{\vec{S}}(N_a - M_a)L^+_{\vec{N}+\vec{M}+\vec{S}}\,, \tag{2.33}$$
$$\{L^-_{\vec{N}}, L^-_{\vec{M}}\} = i\sum_{\vec{S}} \pi^a_{\vec{S}}(N_a - M_a)L^-_{\vec{N}+\vec{M}+\vec{S}}\,, \tag{2.34}$$
$$\{L^+_{\vec{N}}, L^-_{\vec{M}}\} = 0\,, \tag{2.35}$$
$$\{H_{a,\vec{N}}, L^\pm_{\vec{M}}\} = i(N_a - M_a)L^\pm_{\vec{N}+\vec{M}}\,, \tag{2.36}$$
$$\{H_{a,\vec{N}}, H_{b,\vec{M}}\} = iN_b H_{a,\vec{N}+\vec{M}} - iM_a H_{b,\vec{N}+\vec{M}}\,. \tag{2.37}$$

We see that the subalgebra generated by $L^+_{\vec{N}}$ and $L^-_{\vec{N}}$ is an ideal of the whole gauge algebra.

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The subalgebra (2.33) (as well as (2.34)) can be thought of as generalizations of the Virasoro algebra. To illustrate this let us choose π^a to be constant and thus the subalgebra (2.33) to be

$$\{L^+_{\vec{N}}, L^+_{\vec{M}}\} = i\pi^a(N_a - M_a)L^+_{\vec{N}+\vec{M}} \,. \tag{2.38}$$

Generically this algebra contains p copies of the standard Virasoro algebra

$$\begin{aligned} \{L^+_{(n,0,0,\ldots,0)}, L^+_{(m,0,0,\ldots,0)}\} &= i\pi^1(n-m)L^+_{(n+m,0,0,\ldots,0)} \,, \\ \{L^+_{(0,n,0,\ldots,0)}, L^+_{(0,m,0,\ldots,0)}\} &= i\pi^2(n-m)L^+_{(0,n+m,0,\ldots,0)} \,, \\ &\ldots \end{aligned} \tag{2.39}$$

π can be absorbed into a redefinition of the generators to bring these to the standard Virasoro algebra form. The embedding of the Virasoro algebras depends on the relative orientation of π^a in R^p. Thus at the level of the classical gauge algebra we see that there is a string sector of the D-brane at the tachyonic vacuum.

In the quantum theory the algebra (2.33)–(2.37) would have a central extension which should be related to the "electric flux" π^a. Thus a consistent quantization of the system may impose restrictions on the allowed "electric fluxes". Since (2.33)–(2.37) is a standard Lie algebra the classical BRST charge can be constructed and it will have rank one. Therefore in principle one may quantize the system. However, the relation of the BRST charge for (2.33)–(2.37) to the full BRST charge of the system (2.25)–(2.29) remains to be determined. In the next section we propose another way of quantizing the system where an algebra similar to (2.33)–(2.37) appears as the algebra of residual symmetries of the model, after a partial gauge-fixing.

3. Lagrangian analysis

In this section we would like to review the problem from the lagrangian point of view. We learn that all, as is usually the case, results can be obtained from the lagrangian approach without any direct reference to Hamiltonian analysis.

The lagrangian (2.1) is not suited for freezing the tachyon field to its minimum (or taking the zero tension limit). Thus the natural approach is to rewrite the lagrangian (2.1) in a different but classically equivalent form which is appropriate for the limit in. Following calculation in [9] one constructs the following equivalent action for the model

$$\begin{aligned} S = \int d^px \Big[&(E^\alpha_1 E^\beta_1 - E^\alpha_2 E^\beta_2)\gamma_{\alpha\beta} + 2\pi\alpha' E^\alpha_{[2}E^\beta_{1]}F_{\alpha\beta} - \frac{1}{(2E^0_1)^2}T^2_p(V(T))^2 \times \\ &\times \det(\gamma_{ab} + 2\pi\alpha' F_{ab})\Big] \,. \end{aligned} \tag{3.1}$$

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Eliminating E_1^α and E_2^α gives back the "classical" BI action (2.1). To study a D-brane at the tachyonic vacuum we drop the last term in (3.1)

$$S = \int d^p x \, [(E_1^\alpha E_1^\beta - E_2^\alpha E_2^\beta)\gamma_{\alpha\beta} + 4\pi\alpha' \partial_\alpha (E_{[1}^\alpha E_{2]}^\beta) A_\beta] \, . \tag{3.2}$$

This action corresponds to the Hamiltonian constraints (2.11), (2.12) (and may be derived from them).

The action (3.2) gives rise the following equations of motion

$$\partial_\alpha (E_{[1}^\alpha E_{2]}^\beta) = 0, \tag{3.3}$$

$$\gamma_{\alpha\beta} E_1^\beta - 2\pi\alpha' F_{\alpha\beta} E_2^\beta = 0 \, , \tag{3.4}$$

$$\gamma_{\alpha\beta} E_2^\beta - 2\pi\alpha' F_{\alpha\beta} E_1^\beta = 0 \, , \tag{3.5}$$

$$\partial_\alpha [(E_1^\alpha E_1^\beta - E_2^\alpha E_2^\beta)\partial_\beta X^\mu] = 0 \, , \tag{3.6}$$

where for the sake of simplicity we use a flat space-time metric $G_{\mu\nu} = \eta_{\mu\nu}$ (The generalization to a general metric is straightforward). In the gauge $E_1^\alpha = \delta_0^\alpha$ equation (3.3) reduces to

$$\partial_a E_2^a = 0 \, , \partial_0 E_2^a = 0 \, . \tag{3.7}$$

From the action (3.2), the canonical momentum π^a conjugated to A_a is $-2\pi\alpha' E_2^a$ and therefore there is a constraint $E_2^0 = 0$. In the present gauge the $2p$ equations (3.4), (3.5) reduce to p independent equations[3] (constraints)

$$\gamma_{0a} - 2\pi\alpha' F_{ab} E_2^b = 0 \, , \gamma_{00} + E_2^a \gamma_{ab} E_2^b = 0 \, . \tag{3.8}$$

These constraints correspond to residual symmetries left after gauge fixing. Also from the action (3.2) the canonical momentum conjugated to A_a is $-2\pi\alpha' E_2^a$ (i.e. π^a) and that the canonical momentum conjugated to X^μ is $\dot{X}^\mu$ (i.e. P_μ). Thus the constraints (3.8) coincide with those we have discussed previously (up to factor a $2\pi\alpha'$). The equation (3.6) becomes

$$\partial_0^2 X^\mu - E_2^a \partial_a E_2^b \partial_b X^\mu = 0 \, . \tag{3.9}$$

Now we can analyze the solutions of the equations of motion in the given gauge. We see that there are no dynamical equations of motion for the "electric" E_2^a and magnetic F_{ab} fields. There are only a Gauss' law for E_2^a and Bianchi identities for F_{ab}. As a first example, let us take $E_2^a = (E, 0, \ldots, 0)$ with E constant and $F_{ab} = 0$ and make the following ansätz for the solution $X^\mu(x_0, x_1, x_2, \ldots, x_p) = Y^\mu(x_0, x_1) f(x_2, \ldots, x_p)$. Because of (3.8) and (3.9) we see that Y^μ's satisfy the following equations

$$\dot{Y}^\mu Y'_\mu = 0 \, , \qquad \dot{Y}^\mu \dot{Y}_\mu + E^2 Y'^\mu Y'_\mu = 0 \, , \qquad (\partial_0 - E\partial_1)(\partial_0 + E\partial_1) Y^\mu = 0 \, , \tag{3.10}$$

[3] Modulo the linear dependent equations and F_{0a}.

where $\dot{Y}^\mu \equiv \partial_0 Y^\mu$ and $Y'^\mu \equiv \partial_1 Y^\mu$. The function f should satisfy the following $(p-1)$ equations

$$(\dot{Y}^\mu Y_\mu)(f\partial_a f) = 0\,, \qquad a = 2, \ldots, p\,. \tag{3.11}$$

As a result of (3.10) Y^μ can be interpreted as a string solution in the conformal gauge, with tension $|E|$. The equations (3.11) are solved by requiring $f = const.$ Thus the string solutions are completely delocalized in the world-volume coordinates $(x_2, \ldots, x_p)$. In other words, the solution corresponds to a set of strings distributed uniformly in the "transverse directions" $(x_2, \ldots, x_p)$.

If we now take the same "electric" field E_2^a as before but a magnetic field F_{ab} different from zero, then the equation (3.10) stays the same while equation (3.11) gets modified to

$$(\dot{Y}^\mu Y_\mu)(f\partial_a f) = 2\pi\alpha' E F_{a1}\,, \qquad a = 2, \ldots, p\,. \tag{3.12}$$

This equation is equivalent to

$$\partial_a(f^2) = \frac{4\alpha' E}{rv} \int\limits_0^\pi dx_1 F_{a1}\,, \tag{3.13}$$

where we assume that $x_1 \in [0, \pi]$ and $r^2 \equiv \frac{1}{\pi}\int\limits_0^\pi dx_1\, Y^\mu Y_\mu$, $v \equiv \dot{r}$. Writing $F_{a1} = \partial_{[a} A_{1]}$ one may solve this equation explicitly. The solution f of this equation will define how the string world-sheets are distributed in the directions $(x_2, \ldots, x_p)$.

So far we have discussed solutions which have an interpretation as a collection of strings filling the world-volume of the brane. One can also construct solutions which correspond to a single string world-sheet which is completely localized in the transverse directions (e.g. $f = \delta(x_2)\ldots\delta(x_p)$ in the previous example). However these solutions are singular and require highly singular electric or magnetic configurations which may be problematic from a computational point of view. These solutions are limiting cases of regular solutions of the theory (in contemporary parlance, they correspond to the boundary points of the moduli space of solutions.).

The analysis can be extended along similar lines for other configurations of E_1^a and F_{ab}. As result one sees that any classical solution of string theory (Polyakov's action) in conformal gauge can be naturally embedded into the present theory in the given gauge. The details of the embedding are governed by E_2^a and F_{ab}. In a different setup (static gauge) similar results were obtained by Sen [8]. However not all solutions of the classical equations of motions are string-like excitations. For instance, assuming that X^μ is independent of x_1 (for the case $E_2^a = (E, 0, \ldots, 0)$ and $F_{ab} = 0$) we find a gauge fixed tensionless $(p-1)$-brane solution, other ansätzes give point particle solutions etc.

In quantizing the system one may adopt the same approach as in the previous classical consideration, i.e. treat the U(1) degrees of freedom as background fields.

Thus one may choose specific configurations of E_2^a and F_{ab} satisfying Gauss' law and the Bianchi identities and then quantize the system considering only the X^μ as quantum excitations. Within this semiclassical treatment the positive modes of the $(p+1)$ constraints (3.8) should be imposed on the physical states. These constraints generate a closed algebra similar to (2.33)–(2.37) but not the same (special care is needed for (2.36)). The algebra is different because E_2^a and A_a are not regarded as a quantum canonical pair of operators, they are just fixed classical background fields. It is not to be expected that this way of quantizing is reliable in all regimes of the theory. However it might give a first insight into the theory and technically it is straightforward to carry out since the algebra of constraints is a Lie algebra. The case of $E_2^a = 0$, $F_{ab} = 0$ corresponds to a tensionless p-brane and using the BRST approach this has been quantized previously [13] (no restrictions such as critical dimensions were found). The next natural generalization is the case of a non-zero electric field E_a^2 and zero magnetic field F_{ab}. This case should be non-trivial since the Virasoro algebra is contained in the full algebra of constraints.

4. Background fields

In this section we would like to consider two related questions: the vacuum fluctuations and the effects of antisymmetric background fields.

Let us begin with a comment on the role of the B-field in the tachyonic vacuum. The effect of the B-field comes from the replacement of $F_{\alpha\beta}$ by $\mathcal{F}_{\alpha\beta} = \partial_{[\alpha}A_{\beta]} + \partial_a X^\mu B_{\mu\nu}\partial_\beta X^\nu$ in (2.1). In the Hamiltonian formalism this results in a redefinition of the momenta P_μ

$$P_\mu \to P_\mu - B_{\mu\nu}\pi^a\partial_a X^\nu \,. \tag{4.1}$$

With this replacement all expressions in Section 2 are still correct. The new string-like constraints $Q^\pm$ correspond to string constraints in a non-trivial B-field background. Thus the effect of the B-field is rather trivial.

Now let us turn to the RR fields. When the tachyon is frozen at the vacuum all RR background field decouple completely from the theory. In general the tachyon T is a world-volume degree of freedom and has a corresponding kinetic term. Proposals for the effective action including a tachyon kinetic term have been put forward in [14]–[16]. However in the present discussion (and as is often the practice) we ignore the dynamics of the tachyon field itself and consider T as a background field. Hence the action has the form

$$S = T_p\int d^{p+1}xV(T)\sqrt{-det(\gamma_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})} + \mu_p\int C\wedge dT\wedge e^{2\pi\alpha' F} + O((\partial T)^2)\,, \tag{4.2}$$

where C is the sum of RR forms. Assuming the expansion[4] of the tachyon potential around the vacuum

$$V(T) = V(T_0) + \frac{1}{2}V''(T_0)(T - T_0)^2 + O(T^3) \tag{4.3}$$

we rewrite the action as

$$S = \int d^p x\,[(E_1^\alpha E_1^\beta - E_2^\alpha E_2^\beta)\gamma_{\alpha\beta} + 2\pi\alpha' E^\alpha_{[2} E^\beta_{1]} F_{\alpha\beta}] + \mu_p \int C \wedge dT \wedge e^{2\pi\alpha' F} + O((\partial T)^2, T^2)\,. \tag{4.4}$$

We may regard T as a static configuration interpolating from one vacuum to another, like a kink or a vertex. The point is that the energy density of these configurations is localized in some of the spatial world-volume coordinates. In an extreme situation dT is a δ-function along those directions. Thus in the approximation used the effect of the Wess-Zumino term is to insert δ sources on the right hand side of (3.3) and (3.6). In the gauge $E_2^\alpha = \delta_0^\alpha$ the equation (3.8) stays the same (thus the Virasoro subalgebra is still present). Apart from those sources all the discussion in the previous section goes through. The presence of δ-source on the right hand side of (3.7) will allow string-like solutions to end where the δ-source sits (since it is a source for the flux $\partial_a E_2^a = j_a$). There are thus open strings which can end on "planes" localized in some of the spatial world-volume coordinates, i.e. on lower dimensional D-branes.

It is far from obvious that one can drop terms of order $O((\partial T)^2, T^2)$ when discussing fluctuations around the vacuum. Most likely that one cannot freely do so. Nevertheless the above qualitative picture is quite reasonable and agrees with expectations. To study the fluctuations around the vacuum more carefully the tachyon field should be treated as dynamical.

5. Discussion

In the present note we have analyzed the classical effective theory of D-branes at the tachyonic vacuum. We have established the following two facts: the string gauge algebra (the Virasoro algebra) is a subalgebra of the D-brane theory at the tachyonic vacuum and the classical string solutions is a subset of the D-brane solutions in this regime. Thus the Nambu-Goto strings can be embedded into D-branes at the tachyonic vacuum. However, it is not clear to what extent the string picture suffices to describe the D-branes in this regime. In the similar situation when $\alpha' \to \infty$ in the fundamental string the quantum theory describes (collection of) massless particles [17]. By analogy one might expect the quantum theory here to describe (a collection of) strings. In fact, the analogy goes even further than indicated, since constraint algebra of tensionless string also contains an ideal ($P_\mu G^{\mu\nu} P_\nu = 0$).

[4] The expansion (4.3) is not always valid, for instance not for $V(T) = (T+1)e^{-T}$ around $T = \infty$. However in this case the argument still goes through, since as a result of (3.1) the correction to the expansion (4.4) is $O((\partial T)^2, (V(T))^2)$ (i.e. exponentially small around $T = \infty$).

We have emphasized that there are problems with quantizing (3.2), but the study of the gauge algebra leads to a suggestion for quantizing the system treating the U(1) degrees of freedom as background degrees of freedom.

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A. Basic definitions

The Lie derivative of arbitrary tensor density of weight n in the direction of the field N^a, is defined as

$$\begin{aligned}\mathcal{L}_{\vec{N}} T_{a_1 a_2 \dots a_k}{}^{b_1 b_2 \dots b_l} &= N^c \partial_c T_{a_1 a_2 \dots a_k}{}^{b_1 b_2 \dots b_l} + \partial_{a_1} N^c T_{c a_2 \dots a_k}{}^{b_1 b_2 \dots b_l} + \dots - \\ &\quad - \partial_c N^{j_1} T_{a_1 a_2 \dots a_k}{}^{c b_2 \dots b_l} - \dots + n \partial_c N^c T_{a_1 a_2 \dots a_k}{}^{b_1 b_2 \dots b_l} .\end{aligned} \tag{A.1}$$

We are using the following basic Poisson brackets

$$\{A_a(x), \pi^b(y)\} = \delta_a^b \delta^{(p)}(x-y) , \qquad \{X^\mu(x), P_\nu(y)\} = \delta_\nu^\mu \delta^{(p)}(x-y) . \tag{A.2}$$

There is the following action of momentum constraint on the fields

$$\begin{aligned} \{X^\mu, \mathcal{H}_a[N^a]\} &= \mathcal{L}_{\vec{N}} X^\mu , \qquad & \{P_\mu, \mathcal{H}_a[N^a]\} &= \mathcal{L}_{\vec{N}} P_\mu , \\ \{F_{ab}, \mathcal{H}_c[N^c]\} &= \mathcal{L}_{\vec{N}} F_{ab} , \qquad & \{\pi^a, \mathcal{H}_c[N^c]\} &= \mathcal{L}_{\vec{N}} \pi^a - N^a \partial_c \pi^c . \end{aligned} \tag{A.3}$$

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